# Standards for Mathematical Practice: A Close Reading

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and

simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger

students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

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| **Key Ideas** |
| Students are expected to:   * Engage in solving problems. * Explain the meaning of a problem and restate in it their own words. * Analyze given information to develop possible strategies for solving the problem. * Identify and execute appropriate strategies to solve the problem. * Check their answers using a different method, and continually ask “Does this make sense?” * Figure out the right question to be asking, what relevant experience they have, what additional information they might need, and where to start. * Have enough stamina to continue even when progress is hard, but enough flexibility to try alternative approaches when progress seems too hard.   Teachers are expected to:   * Provide time for students to discuss problem solving.   Teaching Tips:   * Do not tell students which parts of their prior knowledge to recall and use. * Do not tell students exactly what question they need to answer nor where to begin. |
| **What would students in my classroom be doing?** |
| Grade 5:   * Solve problems by applying their understanding of operations with whole numbers (especially multidigit division), decimals, and fractions including mixed numbers. * Solve problems related to volume and measurement conversions. * Seek the meaning of a problem and look for efficient ways to represent and solve it. * Check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”   Grade 6:   * Solve problems involving ratios and rates and discuss how they solved them. * Solve real world problems through the application of algebraic and geometric concepts. * Seek the meaning of a problem and look for efficient ways to represent and solve it. * Relate expressions to situations. * Check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”   Grade 7:   * Solve problems involving ratios and rates and discuss how they solved them. * Solve real world problems through the application of algebraic and geometric concepts. * Seek the meaning of a problem and look for efficient ways to represent and solve it. * Check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”   Grade 8:   * Solve real world problems through the application of algebraic and geometric concepts. * Seek the meaning of a problem and look for efficient ways to represent and solve it. * Check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”   Algebra I:   * Reason quantitatively and use units to solve problems. * Interpret the structure of expressions * Create equations that describe numbers or relationships * Understand solving equations as a process of reasoning and explain the reasoning * Seek the meaning of a problem and look for efficient ways to represent and solve it. * Check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?” |
| **Instructional Notes** |
| “Real-life Problems”   * What makes a problem “real” is not the context. * A good puzzle is not only more part of a child’s “real world” than, say, figuring out how much paint is needed for a wall, but a better model of the nature of the thinking that goes with “real” problems: the first task in a crossword puzzle or Sudoku or KenKen® is to figure out where to start. * A satisfying puzzle is one that you don’t know how to solve at first, but can figure out. * Common Core assessments will pose problems that are deliberately designed to be different, to require students to “start by explaining to themselves the meaning of a problem and looking for entry points to its solution.” * Teaching can certainly include focused instruction, but students must also get a chance to tackle problems that they have not been taught explicitly how to solve, as long as they have adequate background to figure out how to make progress. |
| **Sample Tasks/Improving Current Tasks** |
| Another way is to provide, somewhat regularly, problems that ask only for the analysis and not for a numeric “answer.” You can develop such problems by modifying standard word problems. For example, consider this standard problem:  *Eva had 36 green pepper seedlings and 24 tomato seedlings. She planted 48 of them. How many more does she have to plant?*  You might leave off some numbers and ask children how they’d solve the problem if the numbers were known. For example:  *Eva started with 36 green pepper seedlings and some tomato seedlings. She planted 48 of them. If you knew how many tomato seedlings she started with, how could you figure out how many seedlings she still has to plant? (I’d add up all the seedlings and subtract 48.)*  Or, you might keep the original numbers but drop off the question and ask what can be figured out from that information, or what questions can be answered.  *Eva had 36 green pepper seedlings and 24 tomato seedlings. She planted 48 of them. (I could ask “how many seedlings did she start with?” and I could figure out that she started with 60. I could ask how many she didn’t plant, and that would be 12. I could ask what is the smallest number of tomato seedlings she planted! She had to have planted at least 12 of them!)*  These alternative word problems ask children for much deeper analysis than typical ones, and you can invent them yourself, just by modifying word problems you already have. |

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x 2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

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| **Key Ideas** |
| Students are expected to:  Teachers are expected to:  Teaching Tips: |
| **What would students in my classroom be doing?** |
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| **Instructional Notes** |
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| **Sample Tasks/Improving Current Tasks** |
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# Sample Performance Tasks

Evaluate the sample performance task provided for your grade level(s).

1. Which Standards for Mathematical Practice are addressed in the task? Explain how the practices you identified are connected to the task.
2. How could the teacher modify or extend this task to provide further opportunities for deep engagement with these or other math practices?
3. What questions could the teacher ask to help students be more aware of their own employment of these practices?
4. Select one of your current lessons or performance tasks and answer the same three questions above. Modify the task and bring samples of student work to your next grade level meeting. Discuss the extent to which students are illustrating their mastery of these math practices.

# Grade 5 Performance Task

Adapted from New York City Department of Education Common Core Aligned Tasks

Content Standards Addressed:

**5.NF 1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

**5.NF 2** Solve word problems involving addition and subtraction of fractions referring to the same whole,

including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

# Grade 6 Performance Task

From Illustrative Mathematics

Content Standards Addressed:

**6RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, i.e., by reasoning

about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Voting for Three - Version One

1. John, Marie, and Will all ran for 6th grade class president. Of the 36 students, 16 voted for John, 12 for Marie, and 8 for Will. What was the ratio of votes for John to votes for Will? What was the ratio of votes for Marie to votes for Will? What was the ratio of votes for Marie to votes for John?
2. Because no one got half the votes, they had to have a run-off election. Marie dropped out and convinced all her voters to vote for Will. What is the new ratio of Will's votes to John's?
3. John and Will also ran for Middle School Council President. There are 90 students voting in middle school. If the ratio of Will's votes to John's votes remains the same as it was in part (b), how many more votes will Will get than John?

Voting for Three – Version Two

John, Marie, and Will all ran for 6th grade class president. Of the 36 students voting, the ratio of votes for John to votes for Will was two to one. Marie got exactly the average number of votes for the three of them. How many more votes did John get than Marie?

Voting for Three – Version Three

John, Marie, and Will all ran for 6th grade class president. The ratio of votes for John to votes for Will was two to one. Marie got exactly the average number of votes for the three of them. John got more votes than Marie. What fraction of the total votes was this difference?

# Grade 7 Performance Task

From University of Pittsburgh

Content Standards Addressed:

**7.RP.2** Recognize and represent proportional relationships between quantities.

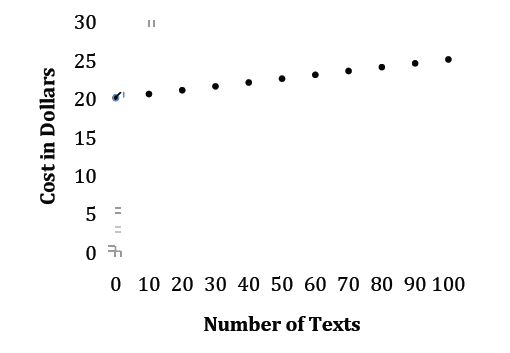
**7.RP.2b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions.

**7.RP.2c** Represent proportional relationships by equations.

**7.RP.2d** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation.

**7.RP.3** Use proportional relationships to solve multistep ratio and percent problems.

The monthly cost of Jazmine’s cell phone plan is graphed on the grid below. Her friend selected a plan that charges $0.25 per text, with no monthly fee, because she only uses her phone for texting.



a. Write an equation to represent the monthly cost of Kiara’s plan for any number of texts.

b. Graph the monthly cost of Kiara’s plan on the grid above.

c. Using the graphs above, explain the meaning of the following coordinate pairs:

i. (0, 20): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

ii. (0, 0): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

iii. (10, 2.5): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

iv. (100, 25): \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

d. When one of the girls double the number of texts she sends, the cost doubles as well. Who is it? Explain in writing how you know.

# Grade 8 Performance Task

From Illustrative Mathematics

Content Standards Addressed:

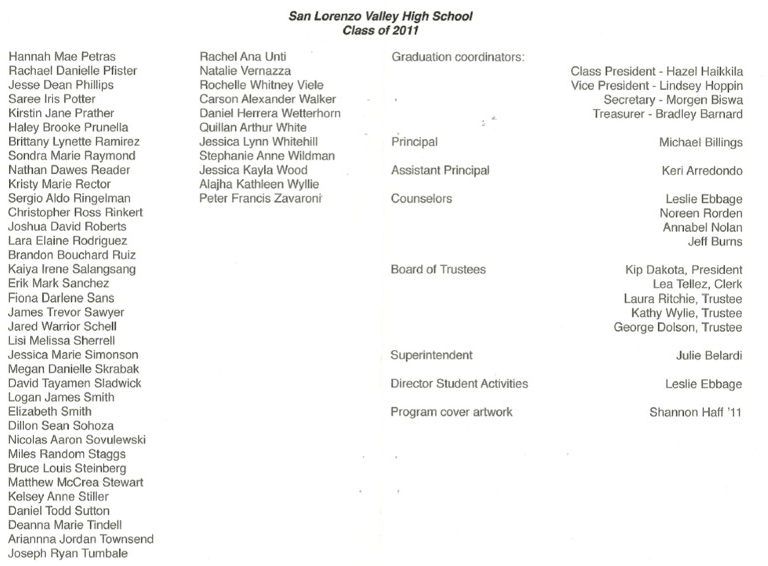
**8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

High School Graduation

The SLV High School graduation started at 1:00 p.m. After some speeches, the principal started reading off the names of the students, alphabetically by last name. When he finishes, the graduation will end.

a. Use the bulletin shown below to estimate when the graduation will end.

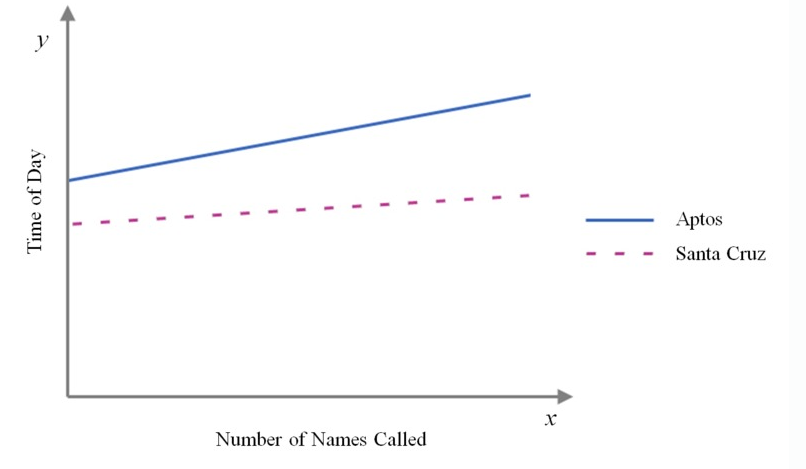




b. Estimate how long the speeches took. How do you know?

c. Write an equation that the parents could use to find the approximate time the principal will call their child’s name given the child’s position in the list in the graduation program.

d. Aptos High School and Santa Cruz High School started their graduations at the same time. The graphs shown below show the time of day as a function of the number of names the principal has read at each school. Write down as many differences between the two graduations as you can based on differences in the two graphs. Give your reasons for each.



# Algebra I Performance Task

From MARS, Shell Centre, University of Nottingham

Content Standards Addressed:

**A-CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions and simple rational and exponential functions.*

**A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**A-REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**A-REI.6**  Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Boomerangs

Phil and Cath make and sell boomerangs for a school event.

The money they raise will go to charity.

They plan to make them in two sizes: small and large.

Phil will carve them from wood.

The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve.

Phil has a total of 24 hours available for carving.

Cath will decorate them.

She only has time to decorate 10 boomerangs of either size.

The small boomerang will make $8 for charity.

The large boomerang will make $10 for charity.

They want to make as much money for charity as they can.

How many small and large boomerangs should they make?

How much money will they then make?